Information Theoretic Concepts of 5G

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Outline

- What is new in 5G
Outline

- What is new in 5G
- Multihop Communications for 5G
Outline

➤ What is new in 5G

➤ Multihop Communications for 5G

➤ Rate-compatible Polar Codes
5G - What is New?

▶ Applications

- Enhanced mobile broadband
- Healthcare
- Traffic control
- Wearables
- Media distribution
- Smart buildings
- Infrastructure
- Industrial processes
5G - What is New?

- Applications
  - Enhanced mobile broadband
  - Healthcare
  - Traffic control
  - Wearables
  - Media distribution
  - Smart buildings
  - Infrastructure
  - Industrial processes

- Requirements
  - 1000x mobile data, 100x user data rates, 100x connected devices, 10x battery life, 5x lower latency
  - Sustainable, secure
5G - What is New?

- Applications
- Requirements
- Architecture - Common network platform
Application Examples

- Broadband experience everywhere anytime
- Mass market personalized TV
- Massive Machine Type Communication
- Critical Machine Type Communication
Machine-type Communications (MTC)

Massive MTC

- Large number, small amount of data, low cost, low energy
Machine-type Communications (MTC)

Massive MTC

- Large number, small amount of data, low cost, low energy

Critical MTC

- Ultra reliable, extremely low latency
New Phy Solutions

- Extension to higher frequencies
New Phy Solutions

- Extension to higher frequencies
- Small cells
New Phy Solutions

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- Massive MIMO
New Phy Solutions

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- Multihop communications
New Phy Solutions

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- Better channel coding
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- Energy-efficient design
New Solutions

- Extension to higher frequencies
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- Energy-efficient design
5G and Spectrum

- Low frequencies: high rates, wide coverage
- mmW band: low complexity, short range

Diagram showing the spectrum with 5G "cellular" access from 3 GHz to 10 GHz and 5G mmW access from 30 GHz to 100 GHz, with LTE compatibility marked.
5G and Spectrum

- Low frequencies: high rates, wide coverage
- mmW band: low complexity, short range
Ultra-dense Networks in mmW Bands

Dense deployments

- Due to limited range
- For higher throughput
Ultra-dense Networks in mmW Bands

Backhaul for thousands of access points?

- Backhaul today: P2P, line-of-sight
- Tomorrow: Wireless multihop backhaul
- Access points relay each other’s data
Ultra-dense Networks in mmW Bands

Backhaul for thousands of access points?

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Efficient multihop communication scheme? What should relays do?
IT: Multihop Setting

What is the capacity region?
Relay Channel

Source $X_s$ → Channel $p(y,y_D|x_s,x)$ → Destination $\hat{W}$

- $W$ to $X_s$
- $X$ from Channel
- $Y$ to $Y_D$
- $\hat{W}$ from Destination

$p(y,y_D|x_s,x)$
THREE-TERMINAL COMMUNICATION CHANNELS

EDWARD C. VAN DER MEUREN, University of Rochester

Summary

The problem of transmitting information in a specified direction over a communication channel with three terminals is considered. Examples are given of the various ways of sending information. Basic inequalities for average mutual information rates are obtained. A coding theorem and weak converse are proved and a necessary and sufficient condition for a positive capacity is derived. Upper and lower bounds on the capacity are obtained, which coincide for channels with symmetric structure.

1. Introduction

In a basic paper Shannon [6] introduced the two-way communication channel and analyzed how to communicate over this channel in two opposite directions as effectively as possible. The present paper considers communication channels which have three different terminals. The problem under investigation is how to send information in one specified direction over such a channel as effectively as possible, assuming that all terminals cooperate so as to optimize the transmission procedure. A three-terminal communication channel is shown schematically in Figure 1. It consists of three terminals, labeled 1, 2, and 3, which are connected to a noisy channel K. At each terminal there is a sender and a receiver who are in direct cross-communication with each other. The sender at one particular terminal may communicate with the receiver at another terminal only through the noisy channel K. The operation of the channel may be described as follows. Once each second, say, at each terminal \( r = 1, 2, \) or \( 3 \), a letter \( x_r \) is selected from a finite set \( A_r \) (the input alphabet at terminal \( r \)) and is presented to the channel for transmission. The channel acts on the input triple \( (x_1, x_2, x_3) \) at once and produces an output triple \( (y_1, y_2, y_3) \). The letter \( y_r \) observed at terminal \( r \) belongs to a finite set \( B_r \), the output alphabet at terminal \( r \). The sender at terminal \( r \) sees the letter \( y_r \) only before he selects the next input letter to be transmitted at his terminal over the channel of the preceding nature of the channel, the output \( (y_1, y_2, y_3) \) depends statistically on the

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REFERENCES


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Capacity Theorems for the Relay Channel

THOMAS M. COVER, FELLOW, IEEE, AND ABBAS A. EL GAMAL, MEMBER, IEEE

Abstract—A relay channel consists of an input \( X \), a relay output \( Y \), a channel input \( X' \), and a relay output \( Y' \). The transmitter transmits an input \( X \) chosen from a finite set \( X \) and a collection of probability distributions \( p(x) \) to the receiver \( Y \). The relay carries the information from \( X \) to \( Y' \).

I. INTRODUCTION

The discrete memoryless relay channel denoted by \( (X' \times X, p(x'|x), p(x|x'), p(y|x), p(y'|x')) \) consists of four finite sets: \( X, X', Y, Y' \), and a collection of probability distributions \( p(x'|x), p(x|x'), p(y|x), p(y'|x') \).

The presented challenging problem is to characterize the capacity of the Gaussian relay channel and certain discrete relay channels evaluated. Finally, an achievable lower bound on the capacity of the general relay channel is established.

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T. M. Cover is with the Departments of Electrical Engineering and Statistics, Stanford University, Stanford, Calif. 94305.

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Multihop Schemes in Practice

- Large body of IT results
Multihop Schemes in Practice

- Large body of IT results
  - Efficient multihop schemes developed; capacity bounds, scaling laws and capacity in special cases determined

5G will deploy multihop communications
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- Not much practical impact
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  - Efficient multihop schemes developed; capacity bounds, scaling laws and capacity in special cases determined
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  - There was no need?
- 5G will deploy multihop communications
Multihop Backhaul for Ultra-dense Networks
Multihop MTC?

70000 tracking devices

9 Gbyte/user/hour

© Getty Images Europe
Our goal: deploy IT approaches in 5G multihop communications
Multihop Communications for 5G
Relaying Schemes

- Decode-and-forward (DF) [Cover & El Gamal, 1979]
- Compress-and-forward [Cover & El Gamal, 1979]
- Quantize-map-forward [Avestimehr et al., 2009]
- Noisy network coding (NNC) [Lim et al., 2011]
- Short-message NNC (SNNC) [Hou and Kramer, 2013]
Relaying Schemes

Decode

Decode-and-forward (DF) [Cover & El Gamal, 1979]

Quantize (Don't decode) ▶

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Relaying Schemes

**Decode**
- Decode-and-forward (DF) \([\text{Cover & El Gamal, 1979}]\)

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Pros and Cons

- **Decode-and-Forward**
  - **Good:** Removes noise
  - **Bad:** Imposes decoding constraints

- **Quantize**
  - **Bad:** Propagation of quantization noise
  - **Good:** No decoding constraints
Pros and Cons

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Quantize

- **Bad**: Propagation of quantization noise
- **Good**: No decoding constraints
NNC/SNNC/CF Relay Encoder

\[ Y \rightarrow \text{Relay} \rightarrow X \]
NNC/SNNC/CF Relay Encoder

\[ Y^n \xrightarrow{\hat{Y}^n} X^n \]

Relay
NNC/SNNC/CF Relay Encoder

\[ Y^n \rightarrow \hat{Y}^n \rightarrow X^n \]

\( \hat{Y}^n (l) \)

\( X^n (l) \)

\[ \ldots \]

\[ \ldots \]
NNC/SNNC/CF Relay Encoder

\[ Y^n \xrightarrow{\hat{Y}^n} X^n \]

\[ \hat{Y}^n(l) \quad X^n(l) \]

\[ Y^n \xrightarrow{\cdots} l \xrightarrow{\cdots} X^n(l) \]
NNC/SNNC/CF Relay Encoder

$Y^n \rightarrow \hat{Y}^n \rightarrow X^n$

Relay

$\hat{Y}^n(l)$

$Y^n \rightarrow \ldots \rightarrow l \rightarrow X^n(l)$

▶ **CF**: adds Wyner-Ziv binning
NNC/SNNC/CF Relay Encoder

In block $b$:

$\hat{Y}^n(l_b, l_{b-1})$

$X^n(l_{b-1})$

$Y^n \rightarrow \cdots \rightarrow l_b \rightarrow l_{b-1} \rightarrow X^n(l_{b-1})$

- **CF**: adds Wyner-Ziv binning
NNC Decoder at the Destination

- In block $b = 1, \ldots, B$

NNC: After $B$ blocks, destination decodes $w$ based on $(x_{Sb}(w), x_{1b}(l_{1b}), \ldots, x_{Kb}(l_{Kb}), \hat{y}_{1b}, \ldots, \hat{y}_{Kb}, y_{Db})$ for $b=1, \ldots, B$.

SNNC: Joint decoding of $(l_{b}, w_{b})$ in each block

CF: Successive decoding of $(l_{b}, w_{b})$
NNC Decoder at the Destination

- In block $b = 1, \ldots, B$

$X_{Sb}(w)$

source

relay $k$

destination

$X_{Db}$

$\hat{y}_1, \ldots, \hat{y}_{K_b}$

for $b=1, \ldots, B$. 

SNNC: Joint decoding of $(l_{b}, w_{b})$ in each block

CF: Successive decoding of $(l_{b}, w_{b})$
NNC Decoder at the Destination

In block \( b = 1, \ldots, B \)

- NNC: After \( B \) blocks, destination decodes \( w \) based on

\[
(x_{Sb}(w), x_{1b}(l_1), \ldots, x_{Kb}(l_b), \hat{y}_{1b}, \ldots, \hat{y}_{Kb}, y_{Db})
\]

for \( b = 1, \ldots, B \).
NNC Decoder at the Destination

- In block $b = 1, \ldots, B$

- NNC: After $B$ blocks, destination decodes $w$ based on

\[
(x_{Sb}(w), x_{1b}(l_1), \ldots, x_{Kb}(l_b), \hat{y}_{1b}, \ldots, \hat{y}_{Kb}, y_{Db})
\]

for $b=1, \ldots, B$.

- SNNC: Joint decoding of $(l_b, w_b)$ in each block

- CF: Successive decoding of $(l_b, w_b)$
NNC & SNNC

- In Gaussian networks achieves constant gap to the multicast capacity
NNC & SNNC

- In Gaussian networks achieves constant gap to the multicast capacity
- Can outperform other schemes
Multihop Backhaul
Implementation: Current Proposal for 5G

Interference-avoidance routing
Implementation: Current Proposal for 5G

Interference-avoidance routing
Implementation: Current Proposal for 5G

Interference-avoidance routing

- Each relay performs decode-and-forward
- Establish routes iteratively

Works well in low interference
Does not work in high interference
Implementation: Current Proposal for 5G

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![Diagram showing relay nodes and routes between sources and destinations]
Implementation: Current Proposal for 5G

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Implementation: NNC Challenges

- Decoder complexity
- Relay selection
- Rate calculation
- Full-duplex assumption
- Channel state information
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Relay Selection
Relay Selection
Relay Selection

source

destination
Relay Selection
Relay Selection
Relay Selection

source -> destination

- source to destination
- source to itself
- destination to source
- destination to itself
Multihop via Half-duplex Relays

- Full-duplex

\[
R = \log(1 + \text{SNR})
\]

"Virtual" full-duplex via half-duplex

Successive relaying + DF

\[
R = \log(1 + \text{SNR})
\]
Multihop via Half-duplex Relays

- Full-duplex

\[ R = \log(1 + \text{SNR}) \]

- DF achieves \( R = \log(1 + \text{SNR}) \)
Multihop via Half-duplex Relays

- Full-duplex

- DF achieves $R = \log(1 + \text{SNR})$

- "Virtual" full-duplex via half-duplex

\[ R = \log(1 + \text{SNR}) \]
Multihop via Half-duplex Relays

- Full-duplex

![](source → \sqrt{SNR} → \sqrt{SNR} → \sqrt{SNR} → \sqrt{SNR} → destination)

- DF achieves $R = \log(1 + SNR)$

- "Virtual" full-duplex via half-duplex

![](source → \sqrt{SNR} → \sqrt{SNR} → \sqrt{SNR} → \sqrt{SNR} → destination)
Multihop via Half-duplex Relays

- Full-duplex

\[ R = \log(1 + \text{SNR}) \]

- DF achieves \( R = \log(1 + \text{SNR}) \)

- ”Virtual” full-duplex via half-duplex
Multihop via Half-duplex Relays

- Full-duplex

\[
\text{source} \xrightarrow{\sqrt{\text{SNR}}} \xrightarrow{\sqrt{\text{SNR}}} \xrightarrow{\sqrt{\text{SNR}}} \xrightarrow{\sqrt{\text{SNR}}} \text{destination}
\]

- DF achieves \( R = \log(1 + \text{SNR}) \)

- "Virtual" full-duplex via half-duplex

\[
\text{source} \xrightarrow{\sqrt{\text{SNR}}} \xrightarrow{\sqrt{\text{SNR}}} \xrightarrow{\sqrt{\text{SNR}}} \text{destination}
\]

- Successive relaying + DF \( R = \log(1 + \text{SNR}) \)
Multihop in Practice

- Full duplex

\[ R \leq \log(1 + \text{SNR}) \]

- "Virtual" full duplex via half-duplex

\[ R \leq \log(1 + \text{SNR}) \]

Inter-relay interference
Symmetric $K$-stage Layered Network

\[ R(K) = \log(1 + \text{SNR}) - K \]

\[ R(K) \geq \log(1 + \text{SNR}) - \log(K + 1) \]

\[ \text{Optimal quantization} \]

\[ Q_k = \sum_{i=1}^{K-(k-1)} \left(1 + \frac{\text{SNR}}{\text{SNR}}\right)^i \]

\[ R(K) \geq \log(1 + \text{SNR}) - \log(K + 1) \]

\[ \text{Rate gap not negligible for large } K \]
Symmetric $K$-stage Layered Network

- QMF, NNC, SNNC with noise level quantization ($Q_k = 1, k = 1, \ldots, K$)

$$R^{(K)} = \log(1 + SNR) - K$$
Symmetric $K$-stage Layered Network

- QMF, NNC, SNNC with noise level quantization
  ($Q_k = 1, k = 1, \ldots, K$)
  \[ R^{(K)} = \log(1 + SNR) - K \]

- Stage-depth quantization [Kolte & Ozgur, 2013]
  ($Q_k = K - (k - 1)$)
  \[ R^{(K)} \geq \log(1 + SNR) - \log(K + 1) \]

- Rate gap not negligible for large $K$
Symmetric $K$-stage Layered Network

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  \[ R^{(K)} \geq \log(1 + SNR) - \log(K + 1) \]

- Optimal quantization [Hong & Caire, 2013]
  ($Q_k = \sum_{i=1}^{K-(k-1)} \left( \frac{1 + SNR}{SNR} \right)^i$)
  \[ R^{(K)} \geq \log(1 + SNR) - \log(K + 1) \]
Symmetric $K$-stage Layered Network

- QMF, NNC, SNNC with noise level quantization ($Q_k = 1, k = 1, \ldots, K$)
  \[ R^{(K)} = \log(1 + \text{SNR}) - K \]

- Stage-depth quantization \cite{kolte2013}
  ($Q_k = K - (k - 1)$)
  \[ R^{(K)} \geq \log(1 + \text{SNR}) - \log(K + 1) \]

- Optimal quantization \cite{hong2013}
  ($Q_k = \sum_{i=1}^{K-(k-1)} (\frac{1+\text{SNR}}{\text{SNR}})^i$)
  \[ R^{(K)} \geq \log(1 + \text{SNR}) - \log(K + 1) \]

- Rate gap not negligible for large $K$
How can we improve performance with a simple scheme?
To Improve Performance: Adaptive Scheme

- Relays with good channels decode-and-forward
- Eliminate noise that is propagated by quantization
- The rest of relays quantize
- No decoding constraint at these nodes
- Each relay can deploy rate splitting
- Enables DF relay to partially cancel interference
- Generalizes adaptive scheme of [Hou & Kramer, 2013]
To Improve Performance: Adaptive Scheme

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- Enables DF relay to partially cancel interference

- Generalizes adaptive scheme of [Hou & Kramer, 2013]
To Reduce Complexity: Successive Decoding

Destination successively decodes messages from different layers

- Does not decrease performance in the considered network

[Hong & Caire, 2013]
Proposed Scheme

- Successive relaying \([\text{Razaei et.al., 2008}]\)
Proposed Scheme

- Successive relaying [Razaei et.al., 2008]
Proposed Scheme

- Successive relaying [Razaei et al., 2008]
- Each relay chooses to decode or quantize
Proposed Scheme

- **Successive relaying** [Razaei et.al., 2008]
- Each relay **chooses** to decode or quantize
- Each relay can deploy **rate splitting**
Proposed Scheme

- Successive relaying [Razaei et al., 2008]
- Each relay chooses to decode or quantize
- Each relay can deploy rate splitting
- Quantization level is optimized → relay performs binning
Proposed Scheme

- Successive relaying [Razaei et.al., 2008]
- Each relay chooses to decode or quantize
- Each relay can deploy rate splitting
- Quantization level is optimized → relay performs binning
- Destination performs successive decoding
Relays

- CF relay $k$: quantization + (Wyner-Ziv) binning
- DF relay: decodes and forwards source message or relay message (bin index)
Destination: Successive Decoding

known interference at the destination

Can reliably decode \( l_5 \) if \( r_5 \leq \log(1 + \text{SNR}) \)

Determine \( \hat{y}_5 \) using the bin index \( l_5 \) and \( x_5 \)

From \( \hat{y}_5 - x_5 = y_5 + \hat{z}_5 - x_5 = x_R(l_1) + z_5 + \hat{z}_5 \) can decode \( l_1 \) if \( r_1 \leq \log(1 + \text{SNR}/(1 + \hat{\sigma}_5^2)) \)

Similarly, can reliably decode \( w_1 \) from \( \hat{y}_1 - x_1 \) if \( r \leq \log(1 + \text{SNR}/(1 + \hat{\sigma}_1^2)) \)
Can reliably decode $l_5$ if $r_5 \leq \log(1 + SNR)$
Destination: Successive Decoding

- Can reliably decode $l_5$ if $r_5 \leq \log(1 + SNR)$
- Determine $\hat{y}_5$ using the bin index $l_5$ and $x_5$
Can reliably decode $l_5$ if $r_5 \leq \log(1 + SNR)$

Determine $\hat{y}_5$ using the bin index $l_5$ and $x_5$

From $\hat{y}_5 - x_5 = y_5 + \hat{z}_5 - x_5 = x_R(l_1) + z_5 + \hat{z}_5$ can decode $l_1$ if

$$r_1 \leq \log(1 + SNR/(1 + \hat{\sigma}_5^2))$$
Can reliably decode $l_5$ if $r_5 \leq \log(1 + \text{SNR})$

determine $\hat{y}_5$ using the bin index $l_5$ and $x_5$

From $\hat{y}_5 - x_5 = y_5 + \hat{z}_5 - x_5 = x_R(l_1) + z_5 + \hat{z}_5$ can decode $l_1$ if

$$r_1 \leq \log(1 + \text{SNR}/(1 + \hat{\sigma}_5^2))$$

Similarly, can reliable decode $w$ from $\hat{y}_1 - x_1$ if

$$r \leq \log(1 + \text{SNR}/(1 + \hat{\sigma}_1^2))$$
Achievable Rate

- Derived closed form solution for DMCs and any configuration
  \([\text{Hong, Marić, Hui & Caire, ISIT 2015}]\)

\[\text{Theorem}\]

For the \(K\)-layer virtual full-duplex relay channel, the achievable rate
is the set of all the pairs \((R_{e}/2, R_{o}/2)\) such that

\[R_{e} \leq C(\alpha^2 \text{SNR}, K), R_{o} \leq C(\beta^2 \text{SNR}, K)\]

where

\[C(x, K) = \log(1 + xK) + 1 \frac{1}{(1 + xK + 1 - xK)}\]
Achievable Rate

- Derived closed form solution for DMCs and any configuration
  \[\text{[Hong, Marić, Hui & Caire, ISIT 2015]}\]
- Special case:

Theorem
For the $K$-layer virtual full-duplex relay channel, the achievable rate is the set of all the pairs $(R_e/2, R_o/2)$ such that

\[
R_e \leq C(\alpha^2 \text{SNR}, K), \quad R_o \leq C(\beta^2 \text{SNR}, K)
\]

where

\[
C(x, K) = \log(1 + x)^{K+1}/((1 + x)^{K+1} - x^{K+1})
\]
Improved Performance

- QMF [Avestimehr et.al, 2009]
  \[ R^{(1)} - R^{(K)} = K - 1 \]

- SNNC-optimized [Hong & Caire, 2013]
  \[ R^{(1)} - R^{(K)} \leq \log(K + 1) - 1 \]

- Adaptive scheme with rate splitting [Hong, Marić, Hui & Caire, ITW 2015]
  \[ R^{(1)} - R^{(K)} \leq \frac{1}{2} \log(K) \]
Performance Gains: Strong Interference Regime
Performance Gains: Weak Interference Regime
Summary

- Adaptive scheme reduces the number of quantizing relays and thus the quantization noise accumulation
Summary

- Adaptive scheme reduces the number of quantizing relays and thus the quantization noise accumulation.

- SNNC with rate splitting reduces interference among relays.
Summary

- Adaptive scheme reduces the number of quantizing relays and thus the quantization noise accumulation.
- SNNC with rate splitting reduces interference among relays.
- Optimized quantization improves scaling to $\log(K)$. 
Summary

- **Adaptive scheme** reduces the number of quantizing relays and thus the quantization noise accumulation

- **SNNC with rate splitting** reduces interference among relays

- **Optimized quantization** improves scaling to $\log(K)$

- **Successive cancellation** can be used in a practical decoder
Summary

- **Adaptive scheme** reduces the number of quantizing relays and thus the quantization noise accumulation.

- **SNNC with rate splitting** reduces interference among relays.

- **Optimized quantization** improves scaling to $\log(K)$.

- **Successive cancellation** can be used in a practical decoder.

- **Relay selection** can be performed via interference-harnessing routing.
Rate-compatible Polar Codes
Why Beyond Turbo Codes?

- LTE deploys turbo codes
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▷ LTE deploys turbo codes

▷ Considerations
  ▷ Performance, processing, memory, decoding throughput, energy-efficiency, rate-compatibility
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- Polar codes have better energy efficiency for large blocklength
Rate-Compatible Codes

- A sequence of codes with different rates generated with the same encoder/decoder structure

NACK
NACK
ACK
TX RX

- Rate-compatible codes can be obtained by puncturing
Rate-Compatible Codes

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How to make Polar codes rate-compatible?
How to make Polar codes rate-compatible?
Punctured Polar Code Performance

Obtaining rate 1/2 punctured polar code from 1/3 mother code

FER

SNR [dB]
What We Know

- Let $C(n_i, R_i, A_i)$ denote a polar code of rate $R_i$ with information set $A_i$ such that $|A_i| = n_i R_i$.
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- Let $W_1 \succeq W_2 \succeq \ldots \succeq W_K$ be a sequence of degraded channels with capacities $R_1 \geq R_2 \geq \ldots \geq R_K$
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- Let $W_1 \succeq W_2 \succeq \ldots \succeq W_K$ be a sequence of degraded channels with capacities $R_1 \geq R_2 \geq \ldots \geq R_K$
- Then [Korada, 2009]
- For a fixed block length $n$, we can construct a family of $K$ polar codes of rates $R_1 \geq R_2 \geq \ldots \geq R_K$ such that

$$A_1 \supseteq A_2 \supseteq \ldots \supseteq A_K$$

\[\text{Diagram:}\]

- $n$
- $A_1$
- $A_2$
- $\cdots$
- $A_K$
What We Want

- Goal: For fixed $k$ information bits, construct a family of $K$ polar codes $C(n_i, R_i, A_i)$ such that

$$n_1 < n_2 < \ldots < n_K$$

and

$$\{x_1, \ldots x_{n_i}\} \supseteq \{x_1, \ldots x_{n_j}\} \text{ for } i > j$$
Code Construction for $R_1$ and $R_2$, $R_1 \geq R_2$

- Choose a polar code $C(n_1, R_1, A_1)$ such that $n_1 = k/R_1$
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![Diagram of code construction](image)
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▶ Decode $D$ first by using decoder for $C(n'_2, R_2, D)$
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- Decoded: $(A_1 \setminus A_2) \cup A_2 = A_1$ at rate $R_2$
Encoder & Decoder

information bits

Divider

Polar encoder
\( C(n_1,R_1,A_1) \)

Polar encoder
\( C(n_2',R_2,D) \)

Receiver

Polar decoder
\( C(n_2',R_2,D) \)

frozen
bits

Polar decoder
\( C(n_1,R_1,A_2) \)
Gains

- Polar code deployed at each step
Gains

- Polar code deployed at each step
- Can be used for HARQ-IR
Summary

Multihop communications

Channel coding
Summary

Multihop communications
  ▶ Practical noisy network coding

Channel coding
  ▶ Polar codes
Thank You!